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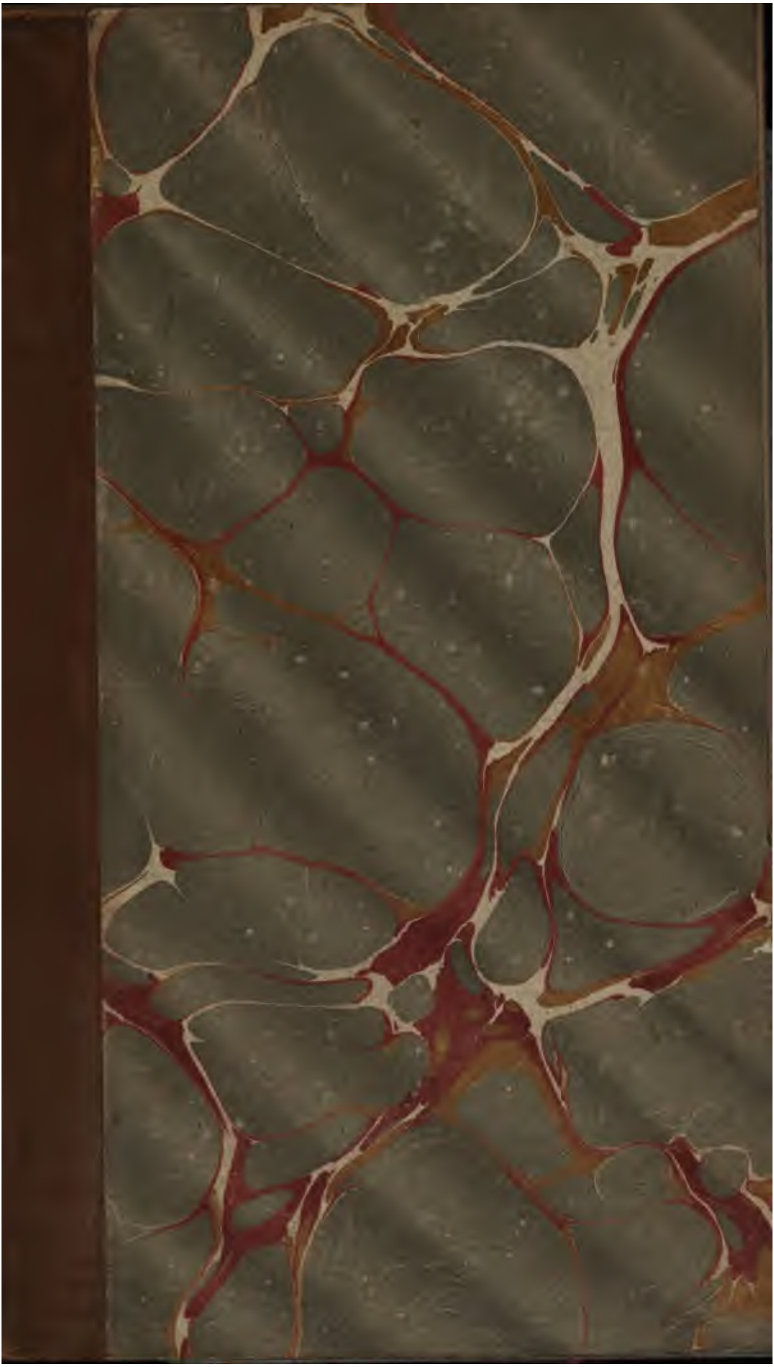

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THE

DEFINITIONS, POSTULATES, AXIOMS, AND ENUNCIATIONS OF THE PROPOSITIONS

OF

THE FIRST SIX

AND

THE ELEVENTH AND TWELFTH BOOKS

OF

EUCLID'S ELEMENTS OF GEOMETRY.

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BOOK I.

DEFINITIONS.

I.

A POINT is that which has no parts, or which has no magnitude.

II.

A line is length without breadth.

III.

The extremities of lines are points.

IV.

A straight line is that which lies evenly between its extreme points.

V.

A superficies is that which has only length and breadth.

VI.

The extremities of superficies are lines.

VII.

A plane superficies is that in which any two points being taken, the straight line between them lies wholly in that superficies.

VIII.

A plane angle is the inclination of two lines to each other in a plane which meet together, but not in the same straight line.

IX.

A plane rectilineal angle is the inclination of two straight lines to one another, which meet together, but are not in the same straight line.

X.

When a straight line standing on another straight line, makes the adjacent angles equal to each other, each of these angles is called a right angle; and the straight line which stands on the other is called a perpendicular to it.

XI.

An obtuse angle is that which is greater than a right angle.

XII.

An acute angle is that which is less than a right angle.

XIII.

A *term* or boundary is the extremity of any thing.

XIV.

A figure is that which is inclosed by one or more boundaries.

XV.

A circle is a plane figure contained by one line, which is called the circumference, and is such that all straight lines drawn from a certain point within the figure to the circumference, are equal to one another.

XVI.

And this point is called the centre of the circle.

XVII.

A diameter of a circle is a straight line drawn through the centre, and terminated both ways by the circumference.

XVIII.

A semicircle is the figure contained by a diameter and the part of the circumference cut off by the diameter.

XIX.

The centre of a semicircle is the same with that of the circle.

XX.

Rectilineal figures are those which are contained by straight lines.

XXI.

Trilateral figures, or triangles, by three straight lines.

XXII.

Quadrilateral, by four straight lines.

XXIII.

Multilateral figures, or polygons, by more than four straight lines.

XXIV.

Of three-sided figures, an equilateral triangle is that which has three equal sides.

XXV.

An isosceles triangle is that which has two sides equal.

XXVI.

A scalene triangle is that which has three unequal sides.

XXVII.

A right-angled triangle is that which has a right angle.

XXVIII.

An obtuse-angled triangle is that which has an obtuse angle.

XXIX.

An acute-angled triangle is that which has three acute angles.

XXX.

Of quadrilateral or four-sided figures, a square has all its sides equal and all its angles right angles.

XXXI.

An oblong is that which has all its angles right angles, but has not all its sides equal.

XXXII.

A rhombus has all its sides equal, but its angles are not right angles.

XXXIII.

A rhomboid has its opposite sides equal to each other, but all its sides are not equal, nor its angles right angles.

XXXIV.

All other four-sided figures besides these, are called Trapeziums.

XXXV.

Parallel straight lines are such as are in the same plane, and which being produced ever so far both ways, do not meet.

A.

A parallelogram is a four-sided figure, of which the opposite sides are parallel: and the diameter or the diagonal is the straight line joining two of its opposite angles.

POSTULATES.

I.

Let it be granted that a straight line may be drawn from any one point to any other point.

II.

That a terminated straight line may be produced to any length in a straight line.

III.

And that a circle may be described from any centre at any distance from that centre.

AXIOMS.

I.

Things which are equal to the same thing are equal to one another.

II.

If equals be added to equals, the wholes are equal.

III.

If equals be taken from equals, the remainders are equal.

IV.

If equals be added to unequals, the wholes are unequal.

V.

If equals be taken from unequals, the remainders are unequal.

VI.

Things which are double of the same, are equal to one another.

VII.

Things which are halves of the same, are equal to one another.

VIII.

Magnitudes which coincide with one another, that is, which exactly fill the same space, are equal to one another.

IX.

The whole is greater than its part.

X.

Two straight lines cannot inclose a space.

XI.

All right angles are equal to one another.

XII.

If a straight line meet two straight lines, so as to make the two interior angles on the same side of it taken together less than two right angles, these straight lines being continually produced, shall at length meet upon that side on which are the angles which are less than two right angles.

PROPOSITIONS.

PROP. I. PROBLEM.

To describe an equilateral triangle upon a given finite straight line.

PROP. II. PROBLEM.

From a given point, to draw a straight line equal to a given straight line.

PROP. III. PROBLEM.

From the greater of two given straight lines, to cut off a part equal to the less.

PROP. IV. THEOREM.

If two triangles have two sides of the one equal to two sides of the other, each to each; and have likewise the angles contained by those sides *equal* to one another; they shall likewise have their bases, or *third sides, equal; and the two triangles* shall be equal; and their other angles shall *be equal, each to each, viz.* those to which the equal sides are opposite.

PROP. V. THEOREM.

The angles at the base of an isosceles triangle are equal to one another: and if the equal sides be produced, the angles upon the other side of the base shall be equal.

Cor. Hence every equilateral triangle is also equiangular.

PROP. VI. THEOREM.

If two angles of a triangle be equal to one another, the sides also which subtend, or *are opposite to*, the equal angles, shall be equal to one another.

Cor. Hence every equiangular triangle is also equilateral.

PROP. VII. THEOREM.

Upon the same base, and on the same side of it, there cannot be two triangles that have their sides which are terminated in one extremity of the base, equal to one another, and likewise those which are terminated in the other extremity.

PROP. VIII. THEOREM.

If two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise their bases equal; the angle which is contained by the two sides of the one shall be equal to the angle contained by the two sides equal to them, of the other.

PROP. IX. PROBLEM.

To bisect a given rectilineal angle, that is, to divide it into two equal angles.

PROP. X. PROBLEM.

To bisect a given finite straight line, that is, to divide it into two equal parts.

PROP. XI. PROBLEM.

To draw a straight line at right angles to a given straight line, from a given point in the same.

Cor. By help of this problem, it may be demonstrated, that two straight lines cannot have a common segment.

PROP. XII. PROBLEM.

To draw a straight line perpendicular to a given straight line of an unlimited length, from a given point without it.

PROP. XIII. THEOREM.

The angles which one straight line makes with another upon one side of it, are either two right angles, or are together equal to two right angles.

PROP. XIV. THEOREM.

If, at a point in a straight line, two other straight lines, upon the opposite sides of it, make the adjacent angles together equal to two right angles, these two straight lines shall be in one and the same straight line.

PROP. XV. THEOREM.

If two straight lines cut one another, the vertical, or opposite angle *shall be equal.*

Cor. 1. From this it is manifest, that, if two straight lines cut one another, the angles which they make at the point where they cut, are together equal to four right angles.

Cor. 2. And consequently, that all the angles made by any number of lines meeting in one point, are together equal to four right angles.

PROP. XVI. THEOREM.

If one side of a triangle be produced, the exterior angle is greater than either of the interior opposite angles.

PROP. XVII. THEOREM.

Any two angles of a triangle are together less than two right angles.

PROP. XVIII. THEOREM.

The greater side of every triangle is opposite to the greater angle.

PROP. XIX. THEOREM.

The greater angle of every triangle is subtended by the greater side, or *has the greater side opposite to it.*

PROP. XX. THEOREM.

Any two sides of a triangle are together greater than the third side.

PROP. XXI. THEOREM.

If from the ends of a side of a triangle, there be drawn two straight lines to a point within the triangle, these shall be less than the other two sides of the triangle, but shall contain a greater angle.

PROP. XXII. PROBLEM.

To make a triangle of which the sides shall be equal to three given straight lines, but any two whatever of these must be greater than the third.

PROP. XXIII. PROBLEM.

At a given point in a given straight line, to make a rectilineal angle equal to a given rectilineal angle.

PROP. XXIV. THEOREM.

If two triangles have two sides of the one equal to two sides of the other, each to each, but the angle contained by the two sides of one of them greater than the angle contained by the two sides equal to them, of the other; the base of that which has the greater angle, shall be greater than the base of the other.

PROP. XXV. THEOREM.

If two triangles have two sides of the one equal to two sides of the other, each to each, but the base of the one greater than the base of the other; the angle contained by the sides of that which has the greater base, shall be greater than the angle contained by the sides, equal to them, of the other.

PROP. XXVI. THEOREM.

If two triangles have two angles of one equal to two angles of the *other, each to each, and* one side equal to one side, viz. either the sides adjacent to the equal angles in each, or the sides opposite to them; then shall the other sides be equal, each to each, and also the third angle of the one to the third angle of the other.

PROP. XXVII. THEOREM.

If a straight line falling upon two other straight lines make the alternate angles equal to one another; these two straight lines shall be parallel.

PROP. XXVIII. THEOREM.

If a straight line falling upon two other straight lines, make the exterior angle equal to the interior and opposite upon the same side of the line; or make the interior angles upon the same side together equal to two right angles; the two straight lines shall be parallel to one another.

PROP. XXIX. THEOREM.

If a straight line fall upon two parallel straight lines, it makes the alternate angles equal to one another; and the exterior angle equal to the interior and opposite upon the same side; and likewise the two interior angles upon the same side together equal to two right angles.

PROP. XXX. THEOREM.

Straight lines which are parallel to the same straight line are parallel to each other.

PROP. XXXI. PROBLEM.

To draw a straight line through a given point parallel to a given straight line.

PROP. XXXII. THEOREM.

If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are together equal to two right angles.

Cor. 1. All the interior angles of any rectilineal figure together with four right angles, are equal to twice as many right angles as the figure has sides.

Cor. 2. All the exterior angles of any rectilineal figure, made by producing the sides successively in the same direction, are together equal to four right angles.

PROP. XXXIII. THEOREM.

The straight lines which join the extremities of two equal and parallel straight lines towards the same parts, are also themselves equal and parallel.

PROP. XXXIV. THEOREM.

The opposite sides and angles of parallelograms are equal to one another, and the diameter *bisects* them, that is, divides them into two equal parts.

PROP. XXXV. THEOREM.

Parallelograms upon the same base, and between the same parallels, are equal to one another.

PROP. XXXVI. THEOREM.

Parallelograms upon equal bases and between the same parallels, are *equal to one another*.

PROP. XXXVII. THEOREM.

Triangles upon the same base and between the same parallels, are equal to one another.

PROP. XXXVIII. THEOREM.

Triangles upon equal bases and between the same parallels, are equal to one another.

PROP. XXXIX. THEOREM.

Equal triangles upon the same base and upon the same side of it, are between the same parallels.

PROP. XL. THEOREM.

Equal triangles upon equal bases, in the same straight line, and towards the same parts, are between the same parallels.

PROP. XLI. THEOREM.

If a parallelogram and a triangle be upon the same base, and between the same parallels; the parallelogram shall be double of the triangle.

PROP. XLII. PROBLEM.

To describe a parallelogram that shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

PROP. XLIII. THEOREM.

The complements of the parallelograms, which are about the diameter of any parallelogram, are equal to one another.

PROP. XLIV. PROBLEM.

To a given straight line to apply a parallelogram, which shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

PROP. XLV. PROBLEM.

To describe a parallelogram equal to a given rectilineal figure, and having an angle equal to a given rectilineal angle.

COR. From this it is manifest how to a given straight line to apply a parallelogram, which shall have an angle equal to a given rectilineal angle, and shall be equal to a given rectilineal figure; *viz.* by applying to the given straight line a parallelogram equal to the first triangle ABD, and having an angle equal to the given angle.

PROP. XLVI. PROBLEM.

To describe a square upon a given straight line.

COR. Hence, every parallelogram that has one right angle, has all its angles right angles.

PROP. XLVII. THEOREM.

In any right-angled triangle, the square which is described upon the side subtending the right angle, is equal to the squares described upon the sides which contain the right angle.

PROP. XLVIII. THEOREM.

If the square described upon one of the sides of a triangle, be equal *to the squares described* upon the other two sides of it; the angle contained *by these two sides is a right* angle.

BOOK II.

DEFINITIONS.

I.

Every right-angled parallelogram is called a *rectangle,* and is said to be contained by any two of the straight lines which contain one of the right angles.

II.

In every parallelogram, any of the parallelograms about a diameter, together with the two complements, is called a gnomon.

PROPOSITIONS.

PROP. I. THEOREM.

If there be two straight lines, one of which is divided into any number of parts; the rectangle contained by the two straight lines, is equal to the rectangles contained by the undivided line, and the several parts of the divided line.

PROP. II. THEOREM.

If a straight line be divided into any two parts, the rectangles contained by the whole and each of the parts, are together equal to the square of the whole line.

PROP. III. THEOREM.

If a straight line be divided into any two parts, the rectangle contained by the whole and one of the parts, is equal to the rectangle contained by the two parts, together with the square of the aforesaid part.

PROP. IV. THEOREM.

If a straight line be divided into any two parts, the square of the whole line is equal to the squares of the two parts, together with twice the rectangle contained by the parts.

Cor. From the demonstration, it is manifest, that the parallelograms about the diameter of a square are likewise squares.

PROP. V. THEOREM.

If a straight line be divided into two equal parts, and also into two unequal parts; the rectangle contained by the unequal parts, together with the square of the line between the points of section, is equal to the square of half the line.

Cor. From this proposition it is manifest, that the difference of the squares of two unequal lines, AC, CD, is equal to the rectangle containe *by their* sum and difference.

PROP. VI. THEOREM.

If a straight line be bisected, and produced to any point; the rectangle contained by the whole line thus produced, and the part of it produced, together with the square of half the line bisected, is equal to the square of the straight line which is made up of the half and the part produced.

PROP. VII. THEOREM.

If a straight line be divided into any two parts, the squares of the whole line, and of one of the parts, are equal to twice the rectangle contained by the whole and that part, together with the square of the other part.

PROP. VIII. THEOREM.

If a straight line be divided into any two parts, four times the rectangle contained by the whole line, and one of the parts, together with the square of the other part, is equal to the square of the straight line, which is made up of the whole and that part.

PROP. IX. THEOREM.

If a straight line be divided into two equal, and also into two unequal parts; the squares of the two unequal parts are together double of the square of half the line, and of the square of the line between the points of section.

PROP. X. THEOREM.

If a straight line be bisected, and produced to any point, the square of the whole line thus produced, and the square of the part of it produced, are together double of the square of half the line bisected, and of the square of the line made up of the half and the part produced.

PROP. XI. PROBLEM.

To divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts, shall be equal to the square of the other part.

PROP. XII. THEOREM.

In obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square of the side subtending the obtuse angle is greater than the squares of the sides containing the obtuse angle, by twice the rectangle contained by the side upon which, when produced, the perpendicular falls, and the straight line intercepted without the triangle between the perpendicular and the obtuse angle.

PROP. XIII. THEOREM.

In every triangle, the square of the side subtending any of the acute angles, is less than the squares of the sides containing that angle, by twice the rectangle contained by either of these sides, and the straight line intercepted between the acute angle and the perpendicular let fall upon it from the opposite angle.

PROP. XIV. PROBLEM.

To describe a square that shall be equal to a given rectilineal figure.

BOOK III.

DEFINITIONS.

I.

Equal circles are those of which the diameters are equal, or from the centres of which the straight lines to the circumferences are equal.

II.

A straight line is said to touch a circle, when it meets the circle, and being produced does not cut it.

III.

Circles are said to touch one another, which meet, but do not cut one another.

IV.

Straight lines are said to be equally distant from the centre of a circle, when the perpendiculars drawn to them from the centre are equal.

V.

And the straight line on which the greater perpendicular falls, is said to be farther from the centre.

VI.

A segment of a circle, is the figure contained by a straight line and the circumference which it cuts off.

VII.

The angle of a segment, is that which is contained by the straight line and the circumference.

VIII.

An angle in a segment, is the angle contained by two straight lines drawn from any point in the circumference of the segment, to the extremities of the straight line which is the base of the segment.

IX.

An angle is said to insist or stand upon the circumference intercepted between the straight lines that contain the angle.

X.

A sector of a circle, is the figure contained by two straight lines drawn from the centre, and the circumference between them.

XI.

Similar segments of circles are those in which the angles are equal, or which contain equal angles.

PROPOSITIONS.

PROP. I. PROBLEM.

To find the centre of a given circle.

Cor. From this it is manifest, that if in a circle a straight line bisect another at right angles, the centre of the circle is in the line which bisects the other.

PROP. II. THEOREM.

If any two points be taken in the circumference of a circle, the straight line which joins them shall fall within the circle.

PROP. III. THEOREM.

If a straight line drawn through the centre of a circle, bisect a straight line in it which does not pass through the centre, it shall cut it at right angles: and conversely, if it cut it at right angles, it shall bisect it.

PROP. IV. THEOREM.

If in a circle two straight lines cut one another, which do not both pass through the centre, they do not bisect each other.

PROP. V. THEOREM.

If two circles cut one another, they shall not have the same centre.

PROP. VI. THEOREM.

If one circle touch another internally, they shall not have the same centre.

PROP. VII. THEOREM.

If any point be taken in the diameter of a circle, which is not the centre, of all the straight lines which can be drawn from it to the circumference, the greatest is that in which the centre is, and the other part of that diameter is the least; and, of any others, that which is nearer to the line which passes through the centre, is always greater than one more remote: and from the same point there can be drawn only two equal straight lines to the circumference, one upon each side of the diameter.

PROP. VIII. THEOREM.

If any point be taken without a circle, and straight lines be draw from it to the circumference, whereof one passes through the centre; of those which fall upon the concave circumference, the greatest is that which passes through the centre; and of the rest, that which is nearer to that through the centre is always greater than the more remote: but of those which fall upon the convex circumference, the least is that between the point without the circle and the diameter; and of the rest, that which is nearer to the least is always less than the more remote: and only two *equal straight lines can be* drawn from the same point to the circumference, *one upon each side of* the line which passes through the centre.

PROP. IX. THEOREM.

If a point be taken within a circle, from which there fall more than two equal straight lines to the circumference, that point is the centre of the circle.

PROP. X. THEOREM.

One circumference of a circle cannot cut another in more than two points.

PROP. XI. THEOREM.

If one circle touch another internally in any point, the straight line which joins their centres being produced shall pass through that point of contact.

PROP. XII. THEOREM.

If two circles touch each other externally in any point, the straight line which joins their centres, shall pass through that point of contact.

PROP. XIII. THEOREM.

One circle cannot touch another in more points than one, whether it touches it on the inside or outside.

PROP. XIV. THEOREM.

Equal straight lines in a circle are equally distant from the centre; and those which are equally distant from the centre, are equal to one another.

PROP. XV. THEOREM.

The diameter is the greatest straight line in a circle; and, of all others, that which is nearer to the centre, is always greater than one more remote: and the greater is nearer to the centre than the less.

PROP. XVI. THEOREM

The straight line drawn at right angles to the diameter of a circle, from the extremity of it, falls without the circle; and no straight line can be drawn from the extremity between that straight line and the circumference, so as not to cut the circle; or, which is the same thing, no straight line can make so great an acute angle with the diameter at its extremity, or so small an angle with the straight line which is at right angles to it, as not to cut the circle.

Cor. From this it is manifest, that the straight line which is drawn at right angles to the diameter of a circle from the extremity of it, touches the circle; and that it touches it only in one point, because, if it did meet the circle in two, it would fall within it. 'Also, it is evident, that there can be but one straight line which touches the circle in the same point.'

PROP. XVII. PROBLEM.

To draw a straight line from a given point, either without or in the circumference, which shall touch a given circle.

PROP. XVIII. THEOREM.

If a straight line touch a circle, the straight line drawn from the centre to the point of contact, shall be perpendicular to the line touching *the circle.*

PROP. XIX. THEOREM.

If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the touching line, the centre of the circle shall be in that line.

PROP. XX. THEOREM.

The angle at the centre of a circle is double of the angle at the circumference upon the same base, that is, upon the same part of the circumference.

PROP. XXI. THEOREM.

The angles in the same segment of a circle are equal to one another.

PROP. XXII. THEOREM.

The opposite angles of any quadrilateral figure inscribed in a circle, are together equal to two right angles.

PROP. XXIII. THEOREM.

Upon the same straight line, and upon the same side of it, there cannot be two similar segments of circles, not coinciding with one another.

PROP. XXIV. THEOREM.

Similar segments of circles upon equal straight lines, are equal to one another.

PROP. XXV. PROBLEM.

A segment of a circle being given, to describe the circle of which it is the segment.

PROP. XXVI. THEOREM.

In equal circles, equal angles stand upon equal circumferences, whether they be at the centres or circumferences.

PROP. XXVII. THEOREM.

In equal circles, the angles which stand upon equal circumferences, are equal to one another, whether they be at the centres or circumferences.

PROP. XXVIII. THEOREM.

In equal circles, equal straight lines cut off equal circumferences, the greater equal to the greater, and the less to the less.

PROP. XXIX. THEOREM.

In equal circles, equal circumferences are subtended by equal straight lines.

PROP. XXX. PROBLEM.

To bisect a given circumference, *that is, to divide it into two equal parts.*

PROP. XXXI. THEOREM.

In a circle, the angle in a semicircle is a right angle; but the angle in a segment greater than a semicircle is less than a right angle; and the angle in a segment less than a semicircle is greater than a right angle.

COR. From this it is manifest, that if one angle of a triangle be equal to the other two, it is a right angle: because the angle adjacent to it is *equal to the same two;* and when the adjacent angles are equal, they are *right angles.*

PROP. XXXII. THEOREM.

If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle; the angles made by this line with the line touching the circle, shall be equal to the angles which are in the alternate segments of the circle.

PROP. XXXIII. PROBLEM.

Upon a given straight line to describe a segment of a circle, which shall contain an angle equal to a given rectilineal angle.

PROP. XXXIV. PROBLEM.

From a given circle, to cut off a segment which shall contain an angle equal to a given rectilineal angle.

PROP. XXXV. THEOREM.

If two straight lines cut one another within a circle, the rectangle contained by the segments of one of them, is equal to the rectangle contained by the segments of the other.

PROP. XXXVI. THEOREM.

If from any point without a circle two straight lines be drawn, one of which cuts the circle, and the other touches it; the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, shall be equal to the square of the line which touches it.

Cor. If from any point without a circle, there be drawn two straight lines cutting it, the rectangles contained by the whole lines and the parts of them without the circle, are equal to one another.

PROP. XXXVII. THEOREM.

If from a point without a circle there be drawn two straight lines, one of which cuts the circle, and the other meets it; if the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, be equal to the square of the line which meets it, the line which meets it shall touch the circle.

2—3

BOOK IV.

DEFINITIONS.

I.

A RECTILINEAL figure is said to be inscribed in another rectilineal figure, when all the angular points of the inscribed figure are upon the sides of the figure in which it is inscribed, each upon each.

II.

In like manner, a figure is said to be described about another figure, when all the sides of the circumscribed figure pass through the angular points of the figure about which it is described, each through each.

III.

A rectilineal figure is said to be inscribed in a circle, when all the angular points of the inscribed figure are upon the circumference of the circle.

IV.

A rectilineal figure is said to be described about a circle, when each side of the circumscribed figure touches the circumference of the circle.

V.

In like manner, a circle is said to be inscribed in a rectilineal figure, when the circumference of the circle touches each side of the figure.

VI.

A circle is said to be described about a rectilineal figure, when the circumference of the circle passes through all the angular points of the figure about which it is described.

VII.

A straight line is said to be placed in a circle, when the extremities of it are in the circumference of the circle.

PROPOSITIONS.

PROP. I. PROBLEM.

IN a given circle, to place a straight line equal to a given straight line not greater than the diameter of a circle.

PROP. II. PROBLEM.

In a given circle, to inscribe a triangle equiangular to a given triangle.

PROP. III. PROBLEM.

About a given circle, to describe a triangle equiangular to a given triangle.

PROP. IV. PROBLEM.

To inscribe a circle in a given triangle.

PROP. V. PROBLEM.

To describe a circle about a given triangle.

Cor. And it is manifest, that when the centre of the circle falls within the triangle, each of its angles is less than a right angle, each of them being in a segment greater than a semicircle: but, when the centre is in one of the sides of the triangle, the angle opposite to this side, being in a semicircle, is a right angle; and, if the centre falls without the triangle, the angle opposite to the side beyond which it is, being in a segment less than a semicircle, is greater than a right angle: wherefore, if the given triangle be acute angled, the centre of the circle falls within it; if it be a right-angled triangle, the centre is in the side opposite to the right angle; and if it be an obtuse-angled triangle, the centre falls without the triangle, beyond the side opposite to the obtuse angle.

PROP. VI. PROBLEM.

To inscribe a square in a given circle.

PROP. VII. PROBLEM.

To describe a square about a given circle.

PROP. VIII. PROBLEM.

To inscribe a circle in a given square.

PROP. IX. PROBLEM.

To describe a circle about a given square.

PROP. X. PROBLEM.

To describe an isosceles triangle, having each of the angles at the base double of the third angle.

PROP. XI. PROBLEM.

To inscribe an equilateral and equiangular pentagon in a given circle.

PROP. XII. PROBLEM.

To describe an equilateral and equiangular pentagon about a given circle.

PROP. XIII. PROBLEM.

To inscribe a circle in a given equilateral and equiangular pentagon.

PROP. XIV. PROBLEM.

To describe a circle about a given equilateral and equiangular pentagon.

PROP. XV. PROBLEM.

To inscribe an equilateral and equiangular hexagon in a given circle.

Cor. From this it is manifest, that the side of the hexagon is equal to the straight line from the centre, that is, to the semidiameter of the circle.

PROP. XVI. PROBLEM.

To inscribe an equilateral and equiangular quindecagon in a given *circle.*

BOOK V.

DEFINITIONS.

I.

A LESS magnitude is said to be *a part* of a greater magnitude, when the less measures the greater; that is, 'when the less is contained a certain number of times exactly in the greater.'

II.

A greater magnitude is said to be a multiple of a less, when the greater is measured by the less, that is, 'when the greater contains the less a certain number of times exactly.'

III.

Ratio is a mutual relation of two magnitudes of the same kind to one another, in respect of quantity.

IV.

Magnitudes are said to have a ratio to one another, when the less can be multiplied so as to exceed the other.

V.

The first of four magnitudes is said to have the same ratio to the second, which the third has to the fourth, when any equimultiples whatsoever of the first and third being taken, and any equimultiples whatsoever of the second and fourth; if the multiple of the first be less than that of the second, the multiple of the third is also less than that of the fourth: or, if the multiple of the first be equal to that of the second, the multiple of the third is also equal to that of the fourth: or, if the multiple of the first be greater than that of the second, the multiple of the third is also greater than that of the fourth.

VI.

Magnitudes which have the same ratio are called proportionals.

'N.B. When four magnitudes are proportionals, it is usually expressed by saying, the first is to the second, as the third to the fourth.'.

VII.

When of the equimultiples of four magnitudes (taken as in the fifth definition), the multiple of the first is greater than that of the second, but the multiple of the third is not greater than the multiple of the fourth; then the first is said to have to the second a greater ratio than the third magnitude has to the fourth: and, on the contrary, the third is said to have to the fourth a less ratio than the first has to the second.

VIII.

Analogy, or proportion, is the similitude of ratios.

IX.

Proportion consists in three terms at least.

X.

When three magnitudes are proportionals, the first is said to have to the third, the duplicate ratio of that which it has to the second.

XI.

When four magnitudes are continual proportionals, the first is said to have to the fourth, the triplicate ratio of that which it has to the second, and so on, quadruplicate, &c. increasing the denomination still by unity, in any number of proportionals.

Definition A, to wit, of compound ratio.

When there are any number of magnitudes of the same kind, the first is said to have to the last of them the ratio compounded of the ratio which the first has to the second, and of the ratio which the second has to the third, and of the ratio which the third has to the fourth, and so on unto the last magnitude.

For example, if A, B, C, D be four magnitudes of the same kind, the first A is said to have to the last D, the ratio compounded of the ratio of A to B, and of the ratio of B to C, and of the ratio of C to D; or, the ratio of A to D is said to be compounded of the ratios of A to B, B to C, and C to D.

And if A has to B the same ratio which E has to F; and B to C the same ratio that G has to H; and C to D the same that K has to L; then, by this definition, A is said to have to D the ratio compounded of ratios which are the same with the ratios of E to F, G to H, and K to L. And the same thing is to be understood when it is more briefly expressed by saying, A has to D the ratio compounded of the ratios of E to F, G to H, and K to L.

In like manner, the same things being supposed, if M has to N the same ratio which A has to D; then, for shortness sake, M is said to have to N the ratio compounded of the ratios of E to F, G to H, and K to L.

XII.

In proportionals, the antecedent terms are called homologous to one another, as also the consequents to one another.

'Geometers make use of the following technical words, to signify certain ways of changing either the order or magnitude of proportionals, so that they continue still to be proportionals.'

XIII.

Permutando, or alternando, by permutation or alternately. This word is used when there are four proportionals, and it is inferred that the first has the same ratio to the third which the second has to the fourth; or that the first is to the third as the second to the fourth: as is shown in Prop. *XVI. of this Fifth Book.*

XIV.

Invertendo, by inversion; when there are four proportionals, and it is inferred, that the second is to the first, as the fourth to the third. Prop. B, Book v.

XV.

Componendo, by composition; when there are four proportionals, and it is inferred that the first together with the second, is to the second, as the third together with the fourth, is to the fourth. Prop. 18, Book v.

XVI.

Dividendo, by division; when there are four proportionals, and it is inferred, that the excess of the first above the second, is to the second, as the excess of the third above the fourth, is to the fourth. Prop. 17, Book v.

XVII.

Convertendo, by conversion; when there are four proportionals, and it is inferred, that the first is to its excess above the second, as the third to its excess above the fourth. Prop. E, Book v.

XVIII.

Ex æquali (sc. distantiâ), or ex æquo, from equality of distance: when there is any number of magnitudes more than two, and as many others, such that they are proportionals when taken two and two of each rank, and it is inferred, that the first is to the last of the first rank of magnitudes, as the first is to the last of the others: 'Of this there are the two following kinds, which arise from the different order in which the magnitudes are taken, two and two.'

XIX.

Ex æquali, from equality. This term is used simply by itself, when the first magnitude is to the second of the first rank, as the first to the second of the other rank; and as the second is to the third of the first rank, so is the second to the third of the other; and so on in order: and the inference is as mentioned in the preceding definition; whence this is called ordinate proportion. It is demonstrated in Prop. 22, Book v.

XX.

Ex æquali in proportione perturbatâ seu inordinatâ, from equality in perturbate or disorderly proportion [*]. This term is used when the first magnitude is to the second of the first rank, as the last but one is to the last of the second rank; and as the second is to the third of the first rank, so is the last but two to the last but one of the second rank; and as the third is to the fourth of the first rank, so is the third from the last to the last but two of the second rank; and so on in a cross order: and the inference is as in the 18th definition. It is demonstrated in Prop. 23, Book v.

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[*] *Prop. 4. Lib. II.* Archimedis de sphærâ et cylindro.

AXIOMS.

I.

Equimultiples of the same, or of equal magnitudes, are equal to one another.

II.

Those magnitudes, of which the same or equal magnitudes are equimultiples, are equal to one another.

III.

A multiple of a greater magnitude is greater than the same multiple of a less.

IV.

That magnitude, of which a multiple is greater than the same multiple of another, is greater than that other magnitude.

PROPOSITIONS.

PROP. I. THEOREM.

If any number of magnitudes be equimultiples of as many, each of each; what multiple soever any one of them is of its part, the same multiple shall all the first magnitudes be of all the other.

PROP. II. THEOREM.

If the first magnitude be the same multiple of the second that the third is of the fourth, and the fifth the same multiple of the second that the sixth is of the fourth; then shall the first together with the fifth be the same multiple of the second, that the third together with the sixth is of the fourth.

Cor. "From this it is plain, that, if any number of magnitudes AB, BG, GH, be multiples of another C; and as many DE, EK, KL, be the same multiples of F, each of each; the whole of the first, *viz.* AH, is the same multiple of C, that the whole of the last, *viz.* DL, is of F."

PROP. III. THEOREM.

If the first be the same multiple of the second, which the third is of the fourth; and if of the first and third there be taken equimultiples; these shall be equimultiples, the one of the second, and the other of the fourth.

PROP. IV. THEOREM.

If the first of four magnitudes has the same ratio to the second which the third has to the fourth; then any equimultiples whatever of the first and third shall have the same ratio to any equimultiples of the second and fourth, *viz.* "the equimultiple of the first shall have the same ratio to that of the second, which the equimultiple of the third has to that of the *fourth.*"

COR. Likewise, if the first has the same ratio to the second, which the third has to the fourth, then also any equimultiples whatever of the first and third have the same ratio to the second and fourth, and in like manner the first and the third have the same ratio to any equimultiples whatever of the second and fourth.

PROP. V. THEOREM.

If one magnitude be the same multiple of another, which a magnitude taken from the first is of a magnitude taken from the other; the remainder shall be the same multiple of the remainder, that the whole is of the whole.

PROP. VI. THEOREM.

If two magnitudes be equimultiples of two others, and, if equimultiples of these be taken from the first two; the remainders are either equal to these others, or equimultiples of them.

PROP. A. THEOREM.

If the first of four magnitudes has to the second the same ratio which the third has to the fourth; then, if the first be greater than the second, the third is also greater than the fourth; and if equal, equal; and if less, less.

PROP. B. THEOREM.

If four magnitudes are proportionals, they are proportionals also when taken inversely.

PROP. C. THEOREM.

If the first be the same multiple of the second, or the same part of it, that the third is of the fourth; the first is to the second, as the third is to the fourth.

PROP. D. THEOREM.

If the first be to the second as the third to the fourth, and if the first be a multiple, or a part of the second; the third is the same multiple, or the same part of the fourth.

PROP. VII. THEOREM.

Equal magnitudes have the same ratio to the same magnitude; and the same has the same ratio to equal magnitudes.

PROP. VIII. THEOREM.

Of two unequal magnitudes, the greater has a greater ratio to any other magnitude than the less has; and the same magnitude has a greater ratio to the less of two other magnitudes, than it has to the greater.

PROP. IX. THEOREM.

Magnitudes which have the same ratio to the same magnitude are equal to one another; and those to which the same magnitude has the same ratio are equal to one another.

PROP. X. THEOREM.

That magnitude which has a greater ratio than another has unto the same magnitude, is the greater of the two: and that magnitude to which *the same has a greater* ratio than it has unto another magnitude, is the *less of the two.*

PROP. XI. THEOREM.

Ratios that are the same to the same ratio, are the same to one another.

PROP. XII. THEOREM.

If any number of magnitudes be proportionals, as one of the antecedents is to its consequent, so shall all the antecedents taken together be to all the consequents.

PROP. XIII. THEOREM.

If the first has to the second the same ratio which the third has to the fourth, but the third to the fourth a greater ratio than the fifth has to the sixth; the first shall also have to the second a greater ratio than the fifth has to the sixth.

COR. And if the first have a greater ratio to the second, than the third has to the fourth, but the third the same ratio to the fourth, which the fifth has to the sixth; it may be demonstrated, in like manner, that the first has a greater ratio to the second, than the fifth has to the sixth.

PROP. XIV. THEOREM.

If the first has to the second, the same ratio which the third has to the fourth; then, if the first be greater than the third, the second shall be greater than the fourth; and if equal, equal; and if less, less.

PROP. XV. THEOREM.

Magnitudes have the same ratio to one another which their equimultiples have.

PROP. XVI. THEOREM.

If four magnitudes of the same kind be proportionals, they shall also be proportionals when taken alternately.

PROP. XVII. THEOREM.

If magnitudes, taken jointly, be proportionals, they shall also be proportionals when taken separately; that is, if two magnitudes together have to one of them, the same ratio which two others have to one of these, the remaining one of the first two shall have to the other the same ratio which the remaining one of the last two has to the other of these.

PROP. XVIII. THEOREM.

If magnitudes, taken separately, be proportionals, they shall also be proportionals when taken jointly; that is, if the first be to the second, as the third to the fourth, the first and second together shall be to the second, as the third and fourth together to the fourth.

PROP. XIX. THEOREM.

If a whole magnitude be to a whole, as a magnitude taken from the first, is to a magnitude taken from the other; the remainder shall be to the remainder, as the whole to the whole.

COR. If the whole be to the whole, as a magnitude taken from the first, is to a magnitude taken from the other; the remainder likewise is to the remainder, as the magnitude taken from the first to that taken from *the other.*

PROP. E. THEOREM.

If four magnitudes be proportionals, they are also proportionals by conversion: that is, the first is to its excess above the second, as the third to its excess above the fourth.

PROP. XX. THEOREM.

If there be three magnitudes, and other three, which, taken two and two, have the same ratio; if the first be greater than the third, the fourth shall be greater than the sixth, and if equal, equal; and if less, less.

PROP. XXI. THEOREM.

If there be three magnitudes, and other three, which have the same ratio taken two and two, but in a cross order; if the first magnitude be greater than the third, the fourth shall be greater than the sixth; and if equal, equal; and if less, less.

PROP. XXII. THEOREM.

If there be any number of magnitudes, and as many others, which, taken two and two in order, have the same ratio; the first shall have to the last of the first magnitudes, the same ratio which the first has to the last of the others.

N.B. *This is usually cited by the words "ex æquali," or "ex æquo."*

PROP. XXIII. THEOREM.

If there be any number of magnitudes, and as many others, which taken two and two, in a cross order, have the same ratio; the first shall have to the last of the first magnitudes the same ratio which the first of the others has to the last.

N.B. *This is usually cited by the words "ex æquali in proportione perturbata;"* or, *"ex æquo perturbato."*

PROP. XXIV. THEOREM.

If the first has to the second the same ratio which the third has to the fourth, and the fifth to the second, the same ratio which the sixth has to the fourth; the first and fifth together shall have to the second, the same ratio which the third and sixth together have to the fourth.

Cor. 1. If the same hypothesis be made as in the proposition, the excess of the first and fifth shall be to the second, as the excess of the third and sixth to the fourth: The demonstration of this is the same with that of the proposition, if division be used instead of composition.

Cor. 2. The proposition holds true of two ranks of magnitudes, whatever be their number, of which each of the first rank has to the second magnitude the same ratio that the corresponding one of the second rank has to a fourth magnitude; as is manifest.

PROP. XXV. THEOREM.

If four magnitudes of the same kind are proportionals, the greatest and least of them together are greater than the other two together.

PROP F. THEOREM.

Ratios which are compounded of the same ratios, are the same with *one another.*

PROP. G. THEOREM.

If several ratios be the same with several ratios, each to each; the ratio which is compounded of ratios which are the same with the first ratios, each to each, is the same with the ratio compounded of ratios, which are the same with the other ratios, each to each.

PROP. H. THEOREM.

If a ratio compounded of several ratios be the same with a ratio compounded of any other ratios, and if one of the first ratios, or a ratio compounded of any of the first, be the same with one of the last ratios, or with the ratio compounded of any of the last; then the ratio compounded of the remaining ratios of the first, or the remaining ratio of the first, if but one remain, is the same with the ratio compounded of those remaining of the last, or with the remaining ratio of the last.

PROP. K. THEOREM.

If there be any number of ratios, and any number of other ratios such, that the ratio compounded of ratios which are the same, with the first ratios, each to each, is the same with the ratio compounded of ratios which are the same, each to each, with the last ratios; and if one of the first ratios, or the ratio which is compounded of ratios which are the same with several of the first ratios, each to each, be the same with one of the last ratios, or with the ratio compounded of ratios which are the same, each to each, with several of the last ratios: then the ratio compounded of ratios which are the same with the remaining ratios of the first, each to each, or the remaining ratio of the first, if but one remain; is the same with the ratio compounded of ratios which are the same with those remaining of the last, each to each, or with the remaining ratio of the last.

BOOK VI.

DEFINITIONS.

I.

Similar rectilineal figures are those which have their several angles equal, each to each, and the sides about the equal angles proportionals.

II.

"Reciprocal figures, viz. triangles and parallelograms are such as have their sides about two of their angles proportionals in such a manner, that a side of the first figure is to a side of the other, as the remaining side of this other is to the remaining side of the first."

III.

A straight line is said to be cut in extreme and mean ratio, when the whole is to the greater segment, as the greater segment is to the less.

IV.

The altitude of any figure, is the straight line drawn from its vertex perpendicular to the base.

PROPOSITIONS.

PROP. I. THEOREM.

Triangles and parallelograms of the same altitude are one to the other as their bases.

Cor. From this it is plain, that triangles and parallelograms that have equal altitudes, are one to another as their bases.

PROP. II. THEOREM.

If a straight line be drawn parallel to one of the sides of a triangle, it shall cut the other sides, or these produced, proportionally: and conversely, if the sides, or the sides produced, be cut proportionally, the straight line which joins the points of section shall be parallel to the remaining side of the triangle.

PROP. III. THEOREM.

If the angle of a triangle be divided into two equal angles, by a straight line which also cuts the base; the segments of the base shall have the same ratio which the other sides of the triangle have to one another: and conversely, if the segments of the base have the same ratio which the other sides of the triangle have to one another; the straight line drawn from the vertex to the point of section, divides the vertical angle into two equal angles.

PROP. A. THEOREM.

If the outward angle of a triangle made by producing one of its sides, be divided into two equal angles, by a straight line which also cuts the base produced; the segments between the dividing line and the extremities of the base, have the same ratio which the other sides of the triangle have to one another: and conversely, if the segments of the base produced, have the same ratio which the other sides of the triangle have, the straight line drawn from the vertex to the point of section, divides the outward angle of the triangle into two equal angles.

PROP. IV. THEOREM.

The sides about the equal angles of equiangular triangles are proportionals; and those which are opposite to the equal angles are homologous sides, that is, are the antecedents or consequents of the ratios.

PROP. V. THEOREM.

If the sides of two triangles, about each of their angles, be proportionals, the triangles shall be equiangular; and the equal angles shall be those which are opposite to the homologous sides.

PROP. VI. THEOREM.

If two triangles have one angle of the one equal to one angle of the other, and the sides about the equal angles proportionals, the triangles shall be equiangular, and shall have those angles equal which are opposite to the homologous sides.

PROP. VII. THEOREM.

If two triangles have one angle of the one equal to one angle of the other, and the sides about two other angles proportionals; then, if each of the remaining angles be either less, or not less, than a right angle, or if one of them be a right angle; the triangles shall be equiangular, and shall have those angles equal about which the sides are proportionals.

PROP. VIII. THEOREM.

In a right-angled triangle, if a perpendicular be drawn from the right angle to the base; the triangles on each side of it are similar to the whole triangle, and to one another.

Cor. From this it is manifest, that the perpendicular drawn from the right angle of a right-angled triangle to the base, is a mean proportional between the segments of the base: and also, that each of the sides is a mean proportional between the base and its segment adjacent to that side.

PROP. IX. PROBLEM.

From a given straight line to cut off any part required.

PROP. X. PROBLEM.

To divide a given straight line similarly to a given divided straight line, that is, into parts that shall have the same ratios to one another which the parts of the divided given straight line have.

PROP. XI. PROBLEM.

To find a third proportional to two given straight lines.

PROP. XII. PROBLEM.

To find a fourth proportional to three given straight lines.

PROP. XIII. PROBLEM.

To find a mean proportional between two given straight lines.

PROP. XIV. THEOREM.

Equal parallelograms, which have one angle of the one equal to one angle of the other, have their sides about the equal angles reciprocally proportional: and conversely, parallelograms that have one angle of the one equal to one angle of the other, and their sides about the equal angles reciprocally proportional, are equal to one another.

PROP. XV. THEOREM.

Equal triangles which have one angle of the one equal to one angle of the other, have their sides about the equal angles reciprocally proportional: and conversely, triangles which have one angle in the one equal to one angle in the other, and their sides about the equal angles reciprocally proportional, are equal to one another.

PROP. XVI. THEOREM.

If four straight lines be proportionals, the rectangle contained by the extremes is equal to the rectangle contained by the means: and conversely, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines are proportionals.

PROP. XVII. THEOREM.

If three straight lines be proportionals, the rectangle contained by the extremes is equal to the square of the mean: and conversely, if the rectangle contained by the extremes be equal to the square of the mean, the three straight lines are proportionals.

PROP. XVIII. PROBLEM.

Upon a given straight line to describe a rectilineal figure similar, and similarly situated, to a given rectilineal figure.

PROP. XIX. THEOREM.

Similar triangles are to one another in the duplicate ratio of their homologous sides.

COR. From this it is manifest, that if three straight lines be proportionals, as the first is to the third, so is any triangle upon the first to a similar and similarly described triangle upon the second.

PROP. XX. THEOREM.

Similar polygons may be divided into the same number of similar triangles, having the same ratio to one another that the polygons have; and the polygons have to one another the duplicate ratio of that which their homologous sides have.

COR. 1. In like manner, it may be proved, that similar four-sided figures, or of any number of sides, are one to another in the duplicate ratio of their homologous sides: and it has already been proved in triangles: therefore, universally, similar rectilineal figures are to one another in the uplicate ratio of their homologous sides.

COR. 2. If three straight lines be proportionals, as the first is to the third, so is any rectilineal figure upon the first, to a similar and similarly described rectilineal figure upon the second.

PROP. XXI. THEOREM.

Rectilineal figures which are similar to the same rectilineal figure, are also similar to one another.

PROP. XXII. THEOREM.

If four straight lines be proportionals, the similar rectilineal figures similarly described upon them shall also be proportionals: and conversely, if the similar rectilineal figures similarly described upon four straight lines be proportionals, those straight lines shall be proportionals.

PROP. XXIII. THEOREM.

Equiangular parallelograms have to one another the ratio which is compounded of the ratios of their sides.

PROP. XXIV. THEOREM.

The parallelograms about the diameter of any parallelogram, are similar to the whole, and to one another.

PROP. XXV. PROBLEM.

To describe a rectilineal figure which shall be similar to one, and equal to another given rectilineal figure.

PROP. XXVI. THEOREM..

If two similar parallelograms have a common angle, and be similarly situated; they are about the same diameter.

PROP. XXVII. THEOREM.

Of all parallelograms applied to the same straight line, and deficient by parallelograms, similar and similarly situated to that which is described upon the half of the line; that which is applied to the half, and is similar to its defect, is the greatest.

PROP. XXVIII. PROBLEM.

To a given straight line, to apply a parallelogram equal to a given rectilineal figure, and deficient by a parallelogram similar to a given parallelogram: but the given rectilineal figure to which the parallelogram to be applied is to be equal, must not be greater than the parallelogram applied to half of the given line, having its defect similar to the defect of that which is to be applied; that is, to the given parallelogram.

PROP. XXIX. PROBLEM.

To a given straight line, to apply a parallelogram equal to a given rectilineal figure, exceeding by a parallelogram similar to another given.

PROP. XXX. PROBLEM.

To cut a given straight line in extreme and mean ratio.

PROP. XXXI. THEOREM.

In right-angled triangles, the rectilineal figure described upon the side opposite to the right angle, is equal to the similar, and similarly described figures upon the sides containing the right angle.

PROP. XXXII. THEOREM.

If two triangles which have two sides of the one proportional to two sides of the other, be joined at one angle, so as to have their homologous sides parallel to one another; the remaining sides shall be in a straight line.

PROP. XXXIII. THEOREM.

In equal circles, angles, whether at the centres or circumferences, have the same ratio which the circumferences on which they stand have to one another: so also have the sectors.

PROP. B. THEOREM.

If an angle of a triangle be bisected by a straight line which likewise cuts the base; the rectangle contained by the sides of the triangle is equal to the rectangle contained by the segments of the base, together with the square of the straight line, which bisects the angle.

PROP. C. THEOREM.

If from any angle of a triangle, a straight line be drawn perpendicular to the base; the rectangle contained by the sides of the triangle, is equal to the rectangle contained by the perpendicular and the diameter of the circle described about the triangle.

PROP. D. THEOREM.

The rectangle contained by the diagonals of a quadrilateral figure inscribed in a circle, is equal to both the rectangles contained by its opposite sides.

BOOK XI.

DEFINITIONS.

I.

A SOLID is that which hath length, breadth, and thickness.

II.

That which bounds a solid is a superficies.

III.

A straight line is perpendicular, or at right angles, to a plane, when it makes right angles with every straight line meeting it in that plane.

IV.

A plane is perpendicular to a plane, when the straight lines drawn in one of the planes perpendicular to the common section of the two planes, are perpendicular to the other plane.

V.

The inclination of a straight line to a plane, is the acute angle contained by that straight line, and another drawn from the point in which the first line meets the plane, to the point in which a perpendicular to the plane drawn from any point of the first line above the plane, meets the same plane.

VI.

The inclination of a plane to a plane, is the acute angle contained by two straight lines drawn from any the same point of their common section at right angles to it, one upon one plane, and the other upon the other plane.

VII.

Two planes are said to have the same, or a like inclination to one another, which two other planes have, when the said angles of inclination are equal to one another.

VIII.

Parallel planes are such as do not meet one another though produced.

IX.

A solid angle is that which is made by the meeting, in one point, of more than two plane angles, which are not in the same plane.

X.

Equal and *similar* solid figures are such as are contained by similar *planes equal in number* and magnitude.

XI.

Similar solid figures are such as have all their solid angles equal, each to each, and are contained by the same number of similar planes.

XII.

A pyramid is a solid figure contained by planes that are constituted betwixt one plane and one point above it in which they meet.

XIII.

A prism is a solid figure contained by plane figures, of which two that are opposite are equal, similar, and parallel to one another; and the others parallelograms.

XIV.

A sphere is a solid figure described by the revolution of a semicircle about its diameter, which remains unmoved.

XV.

The axis of a sphere is the fixed straight line about which the semicircle revolves.

XVI.

The centre of a sphere is the same with that of the semicircle.

XVII.

The diameter of a sphere is any straight line which passes through the centre, and is terminated both ways by the superficies of the sphere.

XVIII.

A cone is a solid figure described by the revolution of a right-angled triangle about one of the sides containing the right angle, which side remains fixed.

If the fixed side be equal to the other side containing the right angle, the cone is called a right-angled cone; if it be less than the other side, an obtuse-angled; and if greater, an acute-angled cone.

XIX.

The axis of a cone is the fixed straight line about which the triangle revolves.

XX.

The base of a cone is the circle described by that side containing the right angle, which revolves.

XXI.

A cylinder is a solid figure described by the revolution of a right-angled parallelogram about one of its sides which remains fixed.

XXII.

The axis of a cylinder is the fixed straight line about which the *parallelogram revolves.*

XXIII.

The bases of a cylinder are the circles described by the two revolving opposite sides of the parallelogram.

XXIV.

Similar cones and cylinders are those which have their axes and the diameters of their bases proportionals.

XXV.

A cube is a solid figure contained by six equal squares.

XXVI.

A tetrahedron is a solid figure contained by four equal and equilateral triangles.

XXVII.

An octahedron is a solid figure contained by eight equal and equilateral triangles.

XXVIII.

A dodecahedron is a solid figure contained by twelve equal pentagons which are equilateral and equiangular.

XXIX.

An icosahedron is a solid figure contained by twenty equal and equilateral triangles.

Def. A.

A parallelopiped is a solid figure contained by six quadrilateral figures, whereof every opposite two are parallel.

PROPOSITIONS.

PROP. I. THEOREM.

One part of a straight line cannot be in a plane, and another part above it.

PROP. II. THEOREM.

Two straight lines which cut one another are in one plane, and three straight lines which meet one another are in one plane.

PROP. III. THEOREM.

If two planes cut one another, their common section is a straight line.

PROP. IV. THEOREM.

If a straight line stand at right angles to each of two straight lines in the point of their intersection, it shall also be at right angles to the plane *which passes through* them, that is, to the plane in which they are.

PROP. V. THEOREM.

If three straight lines meet all in one point, and a straight line stand at right angles to each of them in that point; these three straight lines are in one and the same plane.

PROP. VI. THEOREM.

If two straight lines be at right angles to the same plane, they shall be parallel to one another.

PROP. VII. THEOREM.

If two straight lines be parallel, the straight line drawn from any point in the one to any point in the other, is in the same plane with the parallels.

PROP. VIII. THEOREM.

If two straight lines be parallel, and one of them is at right angles to a plane; the other also shall be at right angles to the same plane.

PROP. IX. THEOREM.

Two straight lines which are each of them parallel to the same straight line, and not in the same plane with it, are parallel to one another.

PROP. X. THEOREM.

If two straight lines meeting one another be parallel to two others that meet one another, and are not in the same plane with the first two; the first two and the other two shall contain equal angles.

PROP. XI. PROBLEM.

To draw a straight line perpendicular to a plane, from a given point above it.

PROP. XII. PROBLEM.

To erect a straight line at right angles to a given plane, from a point given in the plane.

PROP. XIII. THEOREM.

From the same point in a given plane, there cannot be two straight lines at right angles to the plane, upon the same side of it; and there can be but one perpendicular to a plane from a point above the plane.

PROP. XIV. THEOREM.

Planes to which the same straight line is perpendicular, are parallel to one another.

PROP. XV. THEOREM.

If two straight lines meeting one another, be parallel to two straight lines which meet one another, but are not in the same plane with the first two; the plane which passes through these is parallel to the plane passing through the others.

PROP. XVI. THEOREM.

If two parallel planes be cut by another plane, their common sections with it are parallels.

PROP. XVII. THEOREM.

If two straight lines be cut by parallel planes, they shall be cut in the *same ratio.*

PROP. XVIII. THEOREM.

If a straight line be at right angles to a plane, every plane which passes through it shall be at right angles to that plane.

PROP. XIX. THEOREM.

If two planes which cut one another be each of them perpendicular to a third plane; their common section shall be perpendicular to the same plane.

PROP. XX. THEOREM.

If a solid angle be contained by three plane angles, any two of them are greater than the third.

PROP. XXI. THEOREM.

Every solid angle is contained by plane angles, which together are less than four right angles.

PROP. XXII. THEOREM.

If every two of three plane angles be greater than the third, and if the straight lines which contain them be all equal; a triangle may be made of the straight lines that join the extremities of those equal straight lines.

PROP. XXIII. PROBLEM.

To make a solid angle which shall be contained by three given plane angles, any two of them being greater than the third, and all three together less than four right angles.

PROP. A. THEOREM.

If each of two solid angles be contained by three plane angles, which are equal to one another, each to each; the planes in which the equal angles are, have the same inclination to one another.

PROP. B. THEOREM.

If two solid angles be contained, each by three plane angles which are equal to one another, each to each, and alike situated; these solid angles are equal to one another.

PROP. C. THEOREM.

Solid figures which are contained by the same number of equal and similar planes alike situated, and having none of their solid angles contained by more than three plane angles, are equal and similar to one another.

PROP. XXIV. THEOREM.

If a solid be contained by six planes, two and two of which are parallel; the opposite planes are similar and equal parallelograms.

PROP. XXV. THEOREM.

If a solid parallelopiped be cut by a plane parallel to two of its opposite planes; it divides the whole into two solids, the base of one of which shall be to the base of the other, as the one solid is to the other.

PROP. XXVI. PROBLEM.

At a given point in a given straight line, to make a solid angle equal to a given solid angle contained by three plane angles.

PROP. XXVII. PROBLEM.

To describe from a given straight line a solid parallelopiped similar, and similarly situated, to one given.

PROP. XXVIII. THEOREM.

If a solid parallelopiped be cut by a plane passing through the diagonals of two of the opposite planes; it shall be cut in two equal parts.

PROP. XXIX. THEOREM.

Solid parallelopipeds upon the same base, and of the same altitude, the insisting straight lines of which are terminated in the same straight lines in the plane opposite to the base, are equal to one another.

PROP. XXX. THEOREM.

Solid parallelopipeds upon the same base, and of the same altitude, the insisting straight lines of which are not terminated in the same straight lines in the plane opposite to the base, are equal to one another.

PROP. XXXI. THEOREM.

Solid parallelopipeds, which are upon equal bases, and of the same altitude, are equal to one another.

PROP. XXXII. THEOREM.

Solid parallelopipeds which have the same altitude are to one another as their bases.

Cor. From this it is manifest, that prisms upon triangular bases, of the same altitude, are to one another as their bases.

PROP. XXXIII. THEOREM.

Similar solid parallelopipeds are one to another in the triplicate ratio of their homologous sides.

Cor. From this it is manifest, that, if four straight lines be continual proportionals, as the first is to the fourth, so is the solid parallelopiped described from the first to the similar solid similarly described from the second; because the first straight line has to the fourth the triplicate ratio of that which it has to the second.

PROP. D. THEOREM.

Solid parallelopipeds contained by parallelograms equiangular to one another, each to each, that is, of which the solid angles are equal, each to each, have to one another the ratio which is the same with the ratio compounded of the ratios of their sides.

PROP. XXXIV. THEOREM.

The bases and altitudes of equal solid parallelopipeds, are reciprocally proportional: and, conversely, if the bases and altitudes be reciprocally proportional, the solid parallelopipeds are equal.

PROP. XXXV. THEOREM.

If, from the vertices of two equal plane angles, there be drawn two straight lines elevated above the planes in which the angles are, and containing equal angles with the sides of those angles, each to each; and if in the lines above the planes there be taken any points, and from them perpendiculars be drawn to the planes in which the first named angles are; and from the points in which they meet the planes, straight lines be drawn to the vertices of the angles first named; these straight lines shall contain equal angles with the straight lines which are above the planes of the angles.

COR. From this it is manifest, that if from the vertices of two equal plane angles, there be elevated two equal straight lines containing equal angles with the sides of the angles, each to each; the perpendiculars drawn from the extremities of the equal straight lines to the planes of the first angles, are equal to one another.

PROP. XXXVI. THEOREM.

If three straight lines be proportionals, the solid parallelopiped described from all three, as its sides, is equal to the equilateral parallelopiped described from the mean proportional, one of the solid angles of which is contained by three plane angles equal, each to each, to the three plane angles containing one of the solid angles of the other figure.

PROP. XXXVII. THEOREM.

If four straight lines be proportionals, the similar solid parallelopipeds similarly described from them shall also be proportionals: and, conversely, if the similar parallelopipeds similarly described from four straight lines be proportionals, the straight lines shall be proportionals.

PROP. XXXVIII. THEOREM.

"If a plane be perpendicular to another plane, and a straight line be drawn from a point in one of the planes perpendicular to the other plane, this straight line shall fall on the common section of the planes."

PROP. XXXIX. THEOREM.

In a solid parallelopiped, if the sides of two of the opposite planes be divided, each into two equal parts, the common section of the planes passing through the points of division, and the diameter of the solid parallelopiped, cut each other into two equal parts.

PROP. XL. THEOREM.

If there be two triangular prisms of the same altitude, the base of one of which is a parallelogram, and the base of the other a triangle; if the parallelogram be double of the triangle, the prisms shall be equal to one another.

BOOK XII.

PROPOSITIONS.

LEMMA I.

If from the greater of two unequal magnitudes, there be taken more than its half, and from the remainder more than its half; and so on: there shall at length remain a magnitude less than the least of the proposed magnitudes. (Book x. Prop. I.)

PROP. I. THEOREM.

Similar polygons inscribed in circles, are to one another as the squares of their diameters.

PROP. II. THEOREM.

Circles are to one another as the squares of their diameters.

PROP. III. THEOREM.

Every pyramid having a triangular base, may be divided into two equal and similar pyramids having triangular bases, and which are similar to the whole pyramid, and into. two equal prisms which together are greater than half of the whole pyramid.

PROP. IV. THEOREM.

If there be two pyramids of the same altitude, upon triangular bases, and each of them be divided into two equal pyramids similar to the whole pyramid, and also into two equal prisms; and if each of these pyramids be divided in the same manner as the first two, and so on; as the base of one of the first two pyramids is to the base of the other, so shall all the prisms in one of them be to all the prisms in the other, that are produced by the same number of divisions.

PROP. V. THEOREM.

Pyramids of the same altitude which have triangular bases, are to one another as their bases.

PROP. VI. THEOREM.

Pyramids of the same altitude which have polygons for their bases, are to one another as their bases.

PROP. VII. THEOREM.

Every prism having a triangular base may be divided into three pyramids that have triangular bases, and are equal to one another.

Cor. 1. From this it is manifest, that every pyramid is the third part *of a prism* which has the same base, and is of an equal altitude with it: *for if the base of the prism* be any other figure than a triangle, it may be *divided into prisms having* triangular bases.

COR. 2. Prisms of equal altitudes are to one another as their bases; because the pyramids upon the same bases, and of the same altitude, are to one another as their bases.

PROP. VIII. THEOREM.

Similar pyramids, having triangular bases, are one to another in the triplicate ratio of that of their homologous sides.

COR. From this it is evident, that similar pyramids which have multangular bases, are likewise to one another in the triplicate ratio of their homologous sides: for they may be divided into similar pyramids having triangular bases, because the similar polygons, which are their bases, may be divided into the same number of similar triangles homologous to the whole polygons: therefore, as one of the triangular pyramids in the first multangular pyramid is to one of the triangular pyramids in the other, so are all the triangular pyramids in the first to all the triangular pyramids in the other; that is, so is the first multangular pyramid to the other: but one triangular pyramid is to its similar triangular pyramid, in the triplicate ratio of their homologous sides; and therefore the first multangular pyramid has to the other, the triplicate ratio of that which one of the sides of the first has to the homologous side of the other.

PROP. IX. THEOREM.

The bases and altitudes of equal pyramids having triangular bases are reciprocally proportional: and, conversely, triangular pyramids of which the bases and altitudes are reciprocally proportionals, are equal to one another.

PROP. X. THEOREM.

Every cone is the third part of a cylinder which has the same base, and is of an equal altitude with it.

PROP XI. THEOREM.

Cones and cylinders of the same altitude are to one another as their bases.

PROP. XII. THEOREM.

Similar cones and cylinders have to one another the triplicate ratio of that which the diameters of their bases have.

PROP. XIII. THEOREM.

If a cylinder be cut by a plane parallel to its opposite planes, or bases; it divides the cylinder into two cylinders, one of which is to the other as the axis of the first to the axis of the other.

PROP. XIV. THEOREM.

Cones and cylinders upon equal bases are to one another as their altitudes.

PROP. XV. THEOREM.

The bases and altitudes of equal cones and cylinders, are reciprocally proportional; and, conversely, if the bases and altitudes be reciprocally *proportional, the* cones and cylinders are equal to one another.

PROP. XVI. PROBLEM.

In the greater of two circles that have the same centre, to inscribe a polygon of an even number of equal sides, that shall not meet the less circle.

LEMMA II.

If two trapeziums ABCD, EFGH be inscribed in the circles, the centres of which are the points K, L; and if the sides AB, DC be parallel, as also EF, HG; and the other four sides AD, BC, EH, FG, be all equal to one another; but the side AB greater than EF, and DC greater than HG; the straight line KA from the centre of the circle in which the greater sides are, is greater than the straight line LE drawn from the centre to the circumference of the other circle.

PROP. XVII. PROBLEM.

In the greater of two spheres which have the same centre, to describe a solid polyhedron, the superficies of which shall not meet the less sphere.

Cor. And if in the less sphere there be inscribed a solid polyhedron, by drawing straight lines betwixt the points in which the straight lines from the centre of the sphere drawn to all the angles of the solid polyhedron in the greater sphere meet the superficies of the less; in the same order in which are joined the points in which the same lines from the centre meet the superficies of the greater sphere; the solid polyhedron in the sphere BCDE has to this other solid polyhedron the triplicate ratio of that which the diameter of the sphere BCDE has to the diameter of the other sphere. For if these two solids be divided into the same number of pyramids, and in the same order, the pyramids shall be similar to one another, each to each: because they have the solid angles at their common vertex, the centre of the sphere, the same in each pyramid, and their other solid angles at the bases equal to one another, each to each, because they are contained by three plane angles, each equal to each; and the pyramids are contained by the same number of similar planes; and are therefore similar to one another, each to each: but similar pyramids have to one another the triplicate ratio of their homologous sides. Therefore the pyramid of which the base is the quadrilateral KBOS, and vertex A, has to the pyramid in the other sphere of the same order, the triplicate ratio of their homologous sides, that is, of that ratio which AB from the centre of the greater sphere has to the straight line from the same centre to the superficies of the less sphere, and in like manner, each pyramid in the greater sphere has to each of the same order in the less, the triplicate ratio of that which AB has to the semidiameter of the less sphere. And as one antecedent is to its consequent, so are all the antecedents to all the consequents. Wherefore the whole solid polyhedron in the greater sphere has to the whole solid polyhedron in the other, the triplicate ratio of that which AB the semidiameter of the first has to the semidiameter of the other; that is, which the diameter BD of the greater has to the diameter of the other sphere.

PROP. XVIII. THEOREM.

Spheres have to one another the triplicate ratio of that which their diameters have.

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